Çankaya University
Department of Computer Engineering

CENG 277 - Discrete Structures

Name-Surname: 27.11.2014

ID Number:

CLASSWORK 7

We will distribute 29 balls to 3 kids A, B, C such that

- A gets at least 8 balls,
- B gets at least 1 ball,
- C gets at most 10 balls.

In how many different ways can we distribute balls?

Answer:

Represent the number of balls A, B, C gets by the power of x in the following expansions:

$$(x^8 + x^9 + \cdots)(x + x^2 + \cdots)(1 + x + \cdots + x^{10})$$

$$= \frac{x^8}{1-x}\,\frac{x}{1-x}\,\frac{1-x^{11}}{1-x}$$

$$= \frac{x^9 - x^{20}}{(1-x)^3}$$

$$= (x^9 - x^{20}) \sum_{n=0}^{\infty} \binom{n+2}{n} x^n$$

The coefficient of x^{29} is:

$$\binom{22}{20} - \binom{11}{9} = 176$$

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CLASSWORK 7

We will distribute 39 balls to 4 kids A, B, C, D such that

- A gets at most 5 balls,
- B gets at least 1 ball,
- C gets 7 or 8 balls,
- D gets even number of balls.

In how many different ways can we distribute balls?
Answer:

Represent the number of balls A, B, C, D gets by the power of x in the following expansions:

$$(1 + x + \cdots + x^5)(x + x^2 + \cdots)(x^7 + x^8)(1 + x^2 + x^4 + \cdots)$$

$$= \frac{1 - x^6}{1 - x} \frac{x}{1 - x} x^7(1 + x) \frac{1}{1 - x^2}$$

$$= \frac{(x^8 - x^{14})(1 + x)}{(1 - x)^3(1 + x)}$$

$$= \frac{x^8 - x^{14}}{(1 - x)^3}$$

$$= (x^8 - x^{14}) \sum_{n=0}^{\infty} \binom{n + 2}{n} x^n$$

The coefficient of x^{39} is:

$$\binom{33}{31} - \binom{27}{25} = 177$$