



Name-Surname:

04.12.2014

ID Number:

## CLASSWORK 8

We will form 20 letter sequences using the letters  $\{a, b, c, d\}$ . Each letter will be used at least once. How many sequences are possible?

**Answer:**

Using exponential generating functions:

$$\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)^4$$

$$= (e^x - 1)^4$$

$$= e^{4x} - 4e^{3x} + 6e^{2x} - 4e^x + 1$$

Coefficient of  $\frac{x^{20}}{20!}$  is:

$$4^{20} - 4 \cdot 3^{20} + 6 \cdot 2^{20} - 4$$



Name-Surname:

04.12.2014

ID Number:

## CLASSWORK 8

We will form 30 letter sequences using the letters  $\{a, b, c, d, e\}$ . We will use  $b$  at least 3 times. How many sequences are possible?

**Answer:**

Using exponential generating functions:

$$\begin{aligned} & \left( \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right) e^{4x} \\ &= \left( e^x - 1 - x - \frac{x^2}{2!} \right) e^{4x} \\ &= e^{5x} - e^{4x} - xe^{4x} - \frac{x^2}{2!}e^{4x} \end{aligned}$$

Coefficient of  $\frac{x^{30}}{30!}$  is:

$$5^{30} - 4^{30} - 30 \cdot 4^{29} - 435 \cdot 4^{28}$$