Çankaya University
Department of Computer Engineering
2014 - 2015 Fall Semester

CENG 277 - Discrete Structures

First Midterm Examination

1) Consider all permutations of the 29 letter Turkish alphabet $\{A, B, \ldots Z\}$. In how many of these does A come before K and K before Z? (Note that these letters need not be adjacent)

2) There are 80 different types of drinks at the supermarket. There is practically unlimited amount from each type. For your party, you want to buy

a) Exactly 20 bottles.

b) At most 20 bottles.
In how many different ways can you do this?

3) Prove or disprove the following:

a) $((p \wedge \neg r) \vee \neg(q \wedge r)) \iff (\neg r)$

b) Let $x, y \in \mathbb{R}$. $\forall x \forall y((0 < x \;\wedge\; x < y) \quad \Rightarrow \quad x^2 < y^2)$

4) Let $n \in \mathbb{Z}^+$ The expression $2^n + 1$ is divisible by 3 for some values of n. For others, it is not.

a) Guess the correct formula where the expression is always divisible by 3.

b) Prove your guess.

5) In how many different ways can we express 63504 as $63504 = p \cdot q$ where p and q are positive integers and $p < q$?

Answers

1) Choose 3 slots: $\binom{29}{3} = \dfrac{29!}{3!\,26!}$

Place A, B, Z. (In one way). Then, place other 26 letters randomly:

$$\frac{29!}{3!\,26!}\,26! = \frac{29!}{3!}$$

2) **a)** Distribute 20 balls to 80 containers randomly: $\binom{99}{20} = \dfrac{99!}{20!\,79!}$

b) Using the same idea above for all choices from 0 to 20: $\displaystyle\sum_{n=0}^{20} \binom{79+n}{n}$

Second Method: Distribute 20 balls to 81 containers: (81st contains the ones that are not distributed) $\binom{100}{20}$

3) **a)** Make a truth table. $p = 0, q = 0, r = 1$ shows that the claim is wrong.

b) x, y are both positive, so we can multiply $x < y$ and $x < y$ side by side to obtain: $x^2 < y^2$

4) **a)** The claim is: $3 \mid 2^{2n-1} + 1$, for $n = 1, 2, \ldots$

b) Proof:

$n = 1 \quad \Rightarrow \quad 3 \mid 3$. It is correct for base case.

Assume the claim is correct for $n = k$, in other words: $2^{2k-1} + 1 = 3m$.

Now consider $n = k + 1$

$$\begin{aligned} 2^{2k+2-1} + 1 &= 4 \cdot 2^{2k-1} + 1 \\ &= 3 \cdot 2^{2k-1} + 2^{2k-1} + 1 \\ &= 3 \cdot 2^{2k-1} + 3m \\ &= 3p \end{aligned}$$

where p is an integer, so the claim is correct by Mathematical Induction.

5) $63504 = 2^4\, 3^4\, 7^2$

Therefore the total number of factors is: $(4 + 1)(4 + 1)(2 + 1) = 75$.

One of them gives the square root: $252 \cdot 252 = 63504$ which can not be p or q.

The other pairs give one p (smaller factor) and one q so there are

$$\frac{75 - 1}{2} = 37$$

pairs of p, q.